Casimir Forces for Robin Scalar Field on Cylindrical Shell in de Sitter Space

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The Casimir stress on a cylindrical shell in background of conformally flat spacetime for massless scalar field is investigated. In the general case of Robin (mixed) boundary condition, formulae are derived for the vacuum expectation values of the energy–momentum tensor and vacuum forces acting on boundaries. The special case of the dS bulk is considered then different cosmological constants are assumed for the space inside and outside of the shell to have general results applicable to the case of cylindrical domain wall formations in the early universe.

1. INTRODUCTION

The Casimir effect is one of the most interesting manifestations of nontrivial properties of the vacuum state in quantum field theory (Mostepanenko *et al*., 1997; Plunien *et al*., 1986). Since its first prediction by Casimir (1948), this effect has been investigated for different fields having different boundary geometries (Bordag *et al*., 2001; Elizalde, 1995; Elizalde *et al*., 1994; Milton, 1999, 2000). The Casimir effect can be viewed as the polarization of vacuum by boundary conditions or geometry. Therefore, vacuum polarization induced by a gravitational field is also considered as Casimir effect. The types of boundary and conditions that have been most often studied are those associated to well-known problems, e.g. plates, spheres, and vanishing conditions, perfectly conducting conditions, etc. The cylindrical problem with perfectly conducting conditions was first considered in De Raad and Milton (1981), for recent study see Gosdzinsky and Romeo (1998), and Milton (1999).

In the context of hot big bang cosmology, the unified theories of the fundamental interactions predict that the universe passes through a sequence of

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phase transitions. These phase transitions can give rise to domain wall structures determined by the topology of the manifold M of degenerate vacuua (Kibble, 1976; Vilenkin, 1985; Zel'dovich *et al.*, 1975). If M is disconnected, i.e., if $\pi(M)$ is nontrivial, then one can pass from one ordered phase to the other only by going through a domain wall. If M has two connected components, e.g., if there is only a discrete reflection symmetry with $\pi_0(M) = Z_2$, then there will be just two ordered phase separated by a domain wall.

The time evolution of topological defects have played an important role in many branches of physics, e.g., vortices in superconductors (Huebener, 1979) and in superfluid (Donnelly, 1991), defects in liquid crystals (Chandrasekhar and Ranganath, 1986), domain wall (Arodz and Larsen, 1994; Arodz, 1995), cosmic string (Kibble, 1976; Vilenkin, 1985), and a flux tube in QCD (Baker *et al.*, 1991).

Zel'dovich *et al.* (1975) have shown that the energy density of the domain walls is so large that it would dominate the universe completely, violating the observed approximation isotropy and homogeneity. In other words, the domain walls were assumed to somehow disappear again soon after their creation in the early universe, for instance, by collapse, evaporation, or simply by inflating away from our visible universe. Much later, however, Hill *et al.* (1998) introduced the so called light or soft domain walls. They considered a late-time phase transition and found that light domain walls could be produced that were not necessarily in contradiction with observed large-scale structure of the universe. In addition, whatever the cosmological effects, we find it important to obtain a better understanding of the dynamics of domain walls.

Casimir effect in curved spacetime has not been studied extensively. Casimir effect in the presence of a general relativistic domain wall is considered in Setare and Saharian (2001), and a study of the relation between trace anomaly and the Casimir effect can be found in Setare and Rezaeian (2000). Casimir effect may have interesting implications for the early universe. It has been shown, e.g., in Antonsen and Borman (1998) that a closed Robertson–Walker spacetime in which the only contribution to the stress tensor comes from Casimir energy of a scalar field is excluded. In inflationary models, where the dynamics of bubbles may play a major role, this dynamical Casimir effect has not yet been taken into account. Let us mention that in Setare and Mansouri (2001a) we have investigated the Casimir effect of a massless scalar field with Dirichlet boundary condition in spherical shell having different vacuua inside and outside, which represents a bubble in early universe with false/true vacuum inside/outside. In this reference, the sphere has zero thickness. In another paper (Setare, 2001), we have extended the analysis to the spherical shell with nonvanishing thickness. The Casimir stress on two parallel plates with constant comoving distance having different vacua between and outside are calculated in Setare and Mansouri (2001b). In the present paper, we will investigate the vacuum expectation values of the energy–momentum tensor of the conformally coupled scalar field on background of the conformally flat

spacetime. We will consider a cylindrical shell and boundary conditions of the Robin type on the shell. The latter includes the Dirichlet and Neumann boundary conditions as special cases. The Casimir energy–momentum tensor for these geometries can be generated from the corresponding flat spacetime results by using the standard transformation formula. Then we consider cylindrical shell with constant comoving radius having different vacuums inside and outside, i.e., with false/true vacuum inside/outside. Our model may be used to study the effect of the Casimir force on the dynamics of the cylindrical domain wall appearing in the simplest Goldston model. In this model, potential of the scalar field has two equal minima corresponding to degenerate vacuua. Therefore, scalar field maps points at spatial infinity in physical space nontrivially into the vacuum manifold (Vilenkin and Shellard, 1994). Domain wall structure occur at the boundary between these regions of space. One may assume that the outer regions of cylinder are in Λ_{out} vacuum corresponding to degenerate vacuua in domain wall configuration. The Casimir effect for the general Robin boundary conditions on background of the Minkowski spacetime was investigated in Romeo and Saharian (2002) for flat boundaries, and the effect for spherically and cylindrically symmetric boundaries in the case of a general conformal coupling was investigated in Saharian (2001), and Romeo and Saharian (2001). Here we use the results of Romeo and Saharian (2001) to generate vacuum energy–momentum tensor for the cylindrical shell in conformally flat backgrounds. The paper is organized as follows. In the next section the vacuum expectation values of the energy–momentum tensor and vacuum forces acting on shell are evaluated for a general case of a conformally flat background. In Section 3, we study the bulk Casimir effect for a conformal scalar when the bulk is a four-dimensional de Sitter space. Finally, the results are re-mentioned and discussed in last section.

2. VACUUM EXPECTATION VALUES FOR THE ENERGY–MOMENTUM TENSOR

In this paper, we will consider a conformally coupled massless scalar field $\varphi(x)$ satisfying the equation

$$(\nabla_\mu \nabla^\mu + \xi R)\varphi(x) = 0, \quad \xi = \frac{D-1}{4D} \tag{1}$$

on background of a $D + 1$-dimensional conformally flat spacetime with the metric

$$g_{\mu\nu} = e^{-2\sigma(r)}\eta_{\mu\nu}, \quad \mu, \ \nu = 0, 1, \ldots, D. \tag{2}$$

In Equation (1) ∇_μ is the operator of the covariant derivative, and R is the Ricci scalar for the metric $g_{\mu\nu}$. Note that for the metric tensor from Equation (2), one has

$$R = De^{2\sigma}[2\sigma'' - (D-1)\sigma'^2], \tag{3}$$

where the prime corresponds to the differentiation with respect to r. We will assume that the field satisfies the mixed boundary condition

$$(A + Bn^i \nabla_i)\varphi(x) = 0 \qquad (4)$$

on the cylindrical shell with radius a. Here n^i is the normal to the boundary surface, ∇_i is the covariant derivative operator, A and B are constants. The results in the following will depend on the ratio of these coefficients only. However, to keep the transition to the Dirichlet and Neumann cases transparent, we will use the form (4).

It can be shown that for a conformally coupled scalar by using field equation (1), the expression for the energy–momentum tensor can be presented in the form

$$T_{\mu\nu} = \nabla_\mu \varphi \nabla_\nu \varphi - \xi \left[\frac{g_{\mu\nu}}{D-1} \nabla_\rho \nabla^\rho + \nabla_\mu \nabla_\nu + R_{\mu\nu} \right] \varphi^2, \qquad (5)$$

where $R_{\mu\nu}$ is the Ricci tensor. The quantization of a scalar filed on background of metric (2) is standard. Let $\{\varphi_\alpha(x), \varphi_\alpha^*(x)\}$ be a complete set of orthonormalized positive and negative frequency solutions to the field equation (1), obeying boundary condition (4). By expanding the field operator over these eigenfunctions, using the standard commutation rules and the definition of the vacuum state for the vacuum expectation values of the energy–momentum tensor, one obtains

$$\langle 0 | T_{\mu\nu}(x) | 0 \rangle = \sum_\alpha T_{\mu\nu}\{\varphi_\alpha, \varphi_\alpha^*\}, \qquad (6)$$

where $|0\rangle$ is the amplitude for the corresponding vacuum state, and the bilinear form $T_{\mu\nu}\{\varphi, \psi\}$ on the right is determined by the classical energy–momentum tensor (5). In the problem under consideration, we have a conformally trivial situation: conformally invariant field on background of the conformally flat spacetime. Instead of evaluating Equation (6) directly on background of the curved metric, the vacuum expectation values can be obtained from the corresponding flat spacetime results for a scalar field $\bar{\varphi}$ by using the conformal properties of the problem under consideration. Under the conformal transformation $g_{\mu\nu} = \Omega^2 \eta_{\mu\nu}$, the $\bar{\varphi}$ field will change by the rule

$$\varphi(x) = \Omega^{(1-D)/2} \bar{\varphi}(x), \qquad (7)$$

where for metric (2) the conformal factor is given by $\Omega = e^{-\sigma(r)}$. The boundary conditions for the field $\bar{\varphi}(x)$ can be written as follows

$$(\bar{A} + \bar{B}\partial_r)\bar{\varphi} = 0, \qquad (8)$$

with constant Robin coefficients $\bar{A}$ and $\bar{B}$. Comparing to the boundary conditions (4) and taking into account transformation rule (7), we obtain the following

relations between the corresponding Robin coefficients

$$\bar{A} = A + \frac{D-1}{2}\sigma'(a)e^{\sigma(a)}B, \quad \bar{B} = Be^{\sigma(a)}. \tag{9}$$

Note that as Dirichlet boundary conditions are conformally invariant, the Dirichlet scalar in the curved bulk corresponds to the Dirichlet scalar in a flat spacetime. However, for the case of Neumann scalar, the flat spacetime counterpart is a Robin scalar with $\bar{A} = (D-1)\sigma'(a)/2$ and $\bar{B} = 1$. The Casimir effect with boundary conditions (8) on cylindrical shell on background of the Minkowski spacetime is investigated in Romeo and Saharian (2002) for a scalar field with a general conformal coupling parameter. In the case of a conformally coupled scalar, the corresponding regularized VEVs for the energy–momentum tensor have the form

$$\langle 0|T_{\mu\nu}|0\rangle = \mathrm{diag}(\varepsilon, -p_1, -p_2, -p_3, \ldots, -p_D). \tag{10}$$

Here ε is the vacuum energy density, p_1, p_2, $p_3 = p_4 = \cdots = p_D$ are effective pressures in the radial, azimuthal, and longitudinal directions, respectively (vacuum stresses). These quantities are determined by the relations

$$q_{\mathrm{SUB}} = \frac{2^{1-D}\pi^{-(D+1)/2}}{a^{D+1}\Gamma(D/2 - 1/2)}\sum_{n=-\infty}^{+\infty}\int_0^\infty dz\, z^{D+3}\frac{\bar{K}_n(z)}{\bar{I}_n(z)}F_n^{(q)}[I_n(zr/a)], \tag{11}$$

where $I_n(z)$ and $K_n(z)$ are the modified Bessel functions, and

$$F_n^{(\varepsilon)}[f(z)] = \frac{1}{D-1}f^2(z) + \left(2\xi - \frac{1}{2}\right)\left[f'^2(z) + \left(\frac{n^2}{z^2} + 1\right)f^2(z)\right] \tag{12}$$

$$F_n^{(p_1)}[f(z)] = \frac{1}{2}\left[\left(\frac{n^2}{z^2} + 1\right)f^2(z) - f'^2(z)\right] - \frac{2\xi}{z}f(z)f'(z) \tag{13}$$

$$F_n^{(p_2)}[f(z)] = -\left(2\xi - \frac{1}{2}\right)\left[f'^2(z) + \left(\frac{n^2}{z^2} + 1\right)f^2(z)\right]$$

$$+ \frac{2\xi}{z}f(z)f'(z) - \frac{n^2}{z^2}f^2(z) \tag{14}$$

$$F_n^{(p_i)}[f(z)] = -F_n^{(\varepsilon)}[f(z)], \quad i = 3, \ldots, D. \tag{15}$$

Here and later we use the notation

$$\bar{f}(z) \equiv Af(z) + (B/a)zf'(z) \tag{16}$$

for a given function $f(z)$. Similarly, the vacuum expectation values for the exterior of a single cylindrical shell can be obtained, the result is as follows (Romeo and

Saharian, 2002)

$$q_{\text{SUB}} = \frac{2^{1-D}\pi^{-(D+1)/2}}{a^{D+1}\Gamma(D/2-1/2)} \sum_{n=-\infty}^{+\infty} \int_0^\infty dz\, z^{D+3}\, \frac{\bar{I}_n(z)}{\bar{K}_n(z)} F_n^{(q)}[K_n(zr/a)], \quad (17)$$

where we use notations (12)–(15). As we see, these quantities can be obtained from the ones for interior region by the replacements $I \to K$, $K \to I$. Using the expressions for the interior and exterior quantities we have

$$F = \frac{-2^{-D}\pi^{-(D+1)/2}}{a^{D+1}\Gamma(D/2-1/2)} \sum_{n=-\infty}^{+\infty} \int_0^\infty dz\, z^{D+1}$$

$$\times \left[2\beta - 4\xi + (z^2 + n^2 - \beta^2 + 4\xi\beta)\frac{(\tilde{I}_n(z)\tilde{K}_n(z))'}{z\tilde{I}_n'(z)\tilde{K}_n'(z)} \right] \quad (18)$$

for the total vacuum force acting per unit surface of the shell. In these formulae, we have introduced the notation

$$\tilde{f}(z) = z^\beta f(z), \qquad \beta = A/B \quad (19)$$

for a given function $f(z)$.

The vacuum energy–momentum tensor on curved background (2) is obtained by the standard transformation law between conformally related problems (see, for instance, Birrell and Davies, 1986) and has the form

$$\langle T_\nu^\mu[g_{\alpha\beta}]\rangle_{\text{ren}} = \langle T_\nu^\mu[g_{\alpha\beta}]\rangle_{\text{ren}}^{(0)} + \langle T_\nu^\mu[g_{\alpha\beta}]\rangle_{\text{ren}}^{(b)}. \quad (20)$$

Here the first term on the right is the vacuum energy–momentum tensor for the situation without boundaries (gravitational part), and the second one is due to the presence of boundaries. As the quantum field is conformally coupled and the background spacetime is conformally flat, the gravitational part of the energy–momentum tensor is completely determined by the trace anomaly and is related to the divergent part of the corresponding effective action by the relation (Birrell and Davies, 1986)

$$\langle T_\nu^\mu[g_{\alpha\beta}]\rangle_{\text{ren}}^{(0)} = 2g^{\mu\sigma}(x)\frac{\delta}{\delta g^{\nu\sigma}(x)} W_{\text{div}}[g_{\alpha\beta}]. \quad (21)$$

Note that in odd spacetime dimensions, the conformal anomaly is absent and the corresponding gravitational part vanishes:

$$\langle T_\nu^\mu[g_{\alpha\beta}]\rangle_{\text{ren}}^{(0)} = 0, \quad \text{for even } D. \quad (22)$$

The boundary part in Equation (20) is related to the corresponding flat spacetime counterpart (10) by the relation (Birrell and Davies, 1986)

$$\langle T_\nu^\mu[g_{\alpha\beta}]\rangle_{\text{ren}}^{(b)} = \frac{1}{\sqrt{|g|}}\langle \bar{T}_\nu^\mu[\eta_{\alpha\beta}]\rangle_{\text{ren}}. \quad (23)$$

By taking into account Equation (10) from here we obtain

$$\langle T_\nu^\mu[g_{\alpha\beta}]\rangle_{\text{ren}}^{(b)} = e^{(D+1)\sigma(r)}\text{diag}(\varepsilon, -p_1, -p_2, -p_3, \ldots, -p_D), \qquad (24)$$

Now we see that as gravitational part (20) is a continuous function on r, it does not contribute to the forces acting on the boundary and the vacuum force per unit surface acting on the boundary at $r = a$ is determined by the boundary part of the vacuum pressure, $p_D = -\langle T_D^D[g_{\alpha\beta}]\rangle_{\text{ren}}^{(b)}$, taken at the point $r = a$:

$$p_D(a) = e^{(D+1)\sigma(a)}F, \qquad (25)$$

where F is given by (18).

3. CASIMIR STRESS ON CYLINDRICAL SHELL IN DS BACKGROUND

We will consider one of the simplest field-theoretical model in which the domain wall type solutions appear (Vilenkin, 1985). The model involves a single, real-valued scalar field φ with Lagrangian given by

$$L = -1/2 g_{\mu\nu}\partial^\mu\varphi\partial^\nu\varphi - V(\varphi), \qquad (26)$$

and

$$V(\varphi) = \frac{\lambda}{2}(\varphi^2 - v^2)^2, \qquad (27)$$

where λ and v are positive constants. The classical ground states are given by $\varphi = \pm v$. The domain wall arises if there are regions in the space where the field φ has different vacuum values, the domain wall interpolating between such regions. In this paper, we will consider a domain wall between a cylindrical region around z-axis in which $\varphi = \Lambda_{\text{in}}$ and the remaining part of the space where $\varphi = \Lambda_{\text{out}}$.

As an application of the general formulae from Section 2, here we consider the important special case of the dS$_{3+1}$ bulk for which

$$ds^2 = \frac{\alpha^2}{\eta^2}\left[d\eta^2 - \sum_{i=1}^{3}(dx^i)^2\right], \qquad (28)$$

where η is the conformal time

$$-\infty < \eta < 0. \qquad (29)$$

The constant α is related to the cosmological constant as

$$\alpha^2 = \frac{3}{\Lambda}. \qquad (30)$$

Now we consider the pure effect of vacuum polarization due to the gravitational field without any boundary conditions (to see such problem for spherical shell and parallel plate geometry refer to Setare and Mansouri, 2001a, 2001b; Setare, 2001).

The renormalized stress tensor for massless scalar field in de Sitter space is given by Birrell and Davies (1986), and Dowker and Critchley (1976)

$$\langle T_\mu^\nu \rangle = \frac{1}{960\pi^2\alpha^4}\delta_\mu^\nu. \tag{31}$$

The corresponding effective pressure is

$$P = -\langle T_1^1 \rangle = -\langle T_r^r \rangle = -\frac{1}{960\pi^2\alpha^4}, \tag{32}$$

valid for both inside and outside the cylinder. Hence, the effective force on the cylinder due to the gravitational vacuum polarization is zero. Now, assume there are different vacuum inside and outside corresponding to α_{in} and α_{out} for the metric equation (28). Now, the effective pressure created by gravitational part in Equation (32) is different for different part of spacetime

$$P_{in} = -\langle T_r^r \rangle_{in} = -\frac{1}{960\pi^2\alpha_{in}^4} = \frac{-\Lambda_{in}^2}{8640\pi^2}, \tag{33}$$

$$P_{out} = -\langle T_r^r \rangle_{out} = -\frac{1}{960\pi^2\alpha_{out}^4} = \frac{-\Lambda_{out}^2}{8640\pi^2}. \tag{34}$$

Therefore, the gravitational pressure over shell, P_g, is given by

$$P_g = P_{in} - P_{out} = \frac{-1}{8640\pi^2}\left(\Lambda_{in}^2 - \Lambda_{out}^2\right) \tag{35}$$

Now we consider the effective pressure due to the boundary condition under the conformal transformation in four dimensions with the conformal factor given by

$$\Omega(\eta) = \frac{\alpha}{\eta}. \tag{36}$$

The vacuum force acting from inside per unit surface of the cylinder can be found using Equations (11) and (25) for the vacuum radial pressure:

$$F_{in} = \frac{\eta^4}{\alpha_{in}^4}p_1\mid_{r=a-0} = \frac{\eta^4}{\alpha_{in}^4}\frac{1}{4\pi^2a^4}\sum_{n=-\infty}^{+\infty}\int_0^\infty dz\, z^6\frac{\bar{K}_n(z)}{\bar{I}_n(z)}F_n^{(p_1)}[I_n(z)], \tag{37}$$

with notation (13). The expression for the radial projection of the vacuum force acting per unit surface of the cylinder from the outside directly follows from Equations (17) and (25) with $q = p_1$:

$$F_{ext} = -\frac{\eta^4}{\alpha_{out}^4}p_1\mid_{r=a+0} = -\frac{\eta^4}{\alpha_{out}^4}\frac{1}{4\pi^2a^4}\sum_{n=-\infty}^{+\infty}\int_0^\infty dz\, z^6\frac{\bar{I}_n(z)}{\bar{K}_n(z)}F_n^{(p_1)}[K_n(z)], \tag{38}$$

Therefore, the vacuum pressure due to the boundary condition acting on the cylinder is given by

$$P_b = F_{in} + F_{ext} = \frac{\eta^4}{\alpha_{in}^4} \frac{1}{4\pi^2 a^4} \sum_{n=-\infty}^{+\infty} \int_0^\infty dz\, z^6 \frac{\bar{K}_n(z)}{\bar{I}_n(z)} F_n^{(p_1)}[I_n(z)]$$

$$- \frac{\eta^4}{\alpha_{out}^4} \frac{1}{4\pi^2 a^4} \sum_{n=-\infty}^{+\infty} \int_0^\infty dz\, z^6 \frac{\bar{I}_n(z)}{\bar{K}_n(z)} F_n^{(p_1)}[K_n(z)]. \tag{39}$$

The total pressure on the cylinder, P, is then given by

$$P = P_g + P_b = \frac{1}{8640\pi^2}\left(\Lambda_{out}^2 - \Lambda_{in}^2\right) + \frac{\eta^4}{36\pi^2 a^4} \sum_{n=-\infty}^{+\infty} \int_0^\infty dz\, z^6$$

$$\left(\Lambda_{in}^2 \frac{\bar{K}_n(z)}{\bar{I}_n(z)} F_n^{(p_1)}[I_n(z)] - \Lambda_{out}^2 \frac{\bar{I}_n(z)}{\bar{K}_n(z)} F_n^{(p_1)}[K_n(z)]\right) \tag{40}$$

The η- or time dependence of the pressure is intuitively clear due to the time dependence of the physical radius of cylinder. This pressure corresponds to the attractive or repulsive force on the shell if $P < 0$ or $P > 0$. The equilibrium state for the cylinder correspond to the zero values of Equation (40): $p = 0$. Total pressure, may be negative or positive, depending on the difference between the cosmological constant in the two parts of spacetime. Given a false vacuum inside the cylinder and true vacuum outside, i.e., $\Lambda_{in} > \Lambda_{out}$, then the gravitational part is negative and tends to contract the cylinder, but the boundary pressure part may be positive or negative. Therefore, the total effective pressure on the cylinder may be negative, leading to a contraction of the cylinder. The contraction, however, ends for a minimum radius of the cylinder, where both part of the total pressure are equal. For the case of true vacuum inside the cylinder and false vacuum outside, i.e., $\Lambda_{in} < \Lambda_{out}$, the gravitational pressure is positive. In this case, boundary part can be negative or positive depending on the difference between F_{in} and F_{out}. Hence, the total pressure may be either negative or positive.

4. CONCLUSION

In the present paper we have investigated the Casimir effect due to the conformally coupled massless scalar field for a cylindrical shell on background of the conformally flat spacetimes. The general case of the mixed boundary conditions is considered. The vacuum expectation values of the energy–momentum tensor are derived from the corresponding flat spacetime results by using the conformal properties of the problem. Then we consider cylindrical shell with constant comoving radius having different vacuums inside and outside, i.e., with false/true vacuum inside/outside. The boundary induced part for the vacuum energy–momentum

tensor is given by Equation (24), and the corresponding vacuum forces acting per unit surface of the shell have the form of Equations (37) and (38). The effective vacuum pressure due to the boundary condition acting on the cylinder is given by Equation (37). The vacuum polarization due to the gravitational field without any boundary conditions is given by Equation (31), the corresponding gravitational pressure part has the form of Equation (32), which is the same from both sides of the shell, and hence leads to zero effective force. However, when we consider different cosmological constants for the space between and outside of the shell, in this case the effective pressure created by gravitational part is different for different part of the spacetime and add to the boundary part pressure. The total pressure is given by Equation (40). Our calculation shows that for the cylindrical shell with false vacuum inside and true vacuum outside, $\Lambda_{in} > \Lambda_{out}$, the gravitational pressure part is negative but the boundary pressure part may be positive or negative. In contrast, for the case of true vacuum inside the cylinder and false vacuum outside, $\Lambda_{out} > \Lambda_{in}$, the gravitational pressure is positive and boundary part can be negative or positive depending on the difference between F_{in} and F_{out} in Equation (40). Therefore, the detail dynamics of the cylindrical shell depends on different parameters and all cases of contraction and expansion may appear. The result may be of interest in the case of formation of the cosmic cylindrical domain walls in early universe.

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